RADIATIVE HEAT FLUX ABSORBED BY A FLUIDIZED BED IN RADIATIVE HEATING

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An equation is derived for the heat flux absorbed by a fluidized bed in radiative heating.

Working from the analogy between the heating of a solid and the heating of a fluidized bed, we note that since the temperature drop over the height of the bed is small in the case of well-developed boiling, while λ_{ef} is always large [1, 2], the Biot number for the fluidized bed is usually less than 0.25, so that the fluidized bed can be classified as a "thin" object [3]. For thin objects and ordinary values of Δt_{extr}, the temperature drop within the object during the heating is slight, the object is heated uniformly over its thickness (over the height, in the case of a fluidized bed), and the internal heat transfer in many technological processes thus does not limit the heating process.*

In the radiative heating of a fluidized bed, the decisive role is thus played by radiative heat transfer between the radiator and the heat-absorbing surface (the surface of the fluidized bed).

* This assertion is correct except for the case of highly endothermic processes.

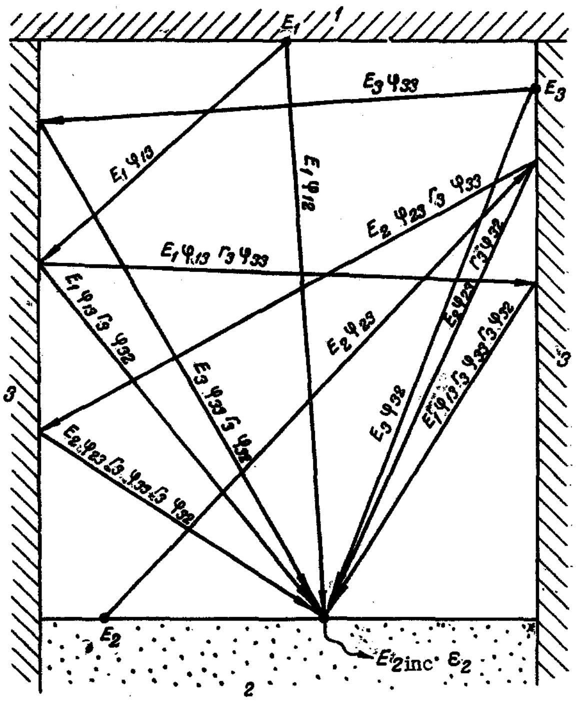

Fig. 1. Diagram used in deriving the equation for the resultant thermal radiation flux absorbed by the fluidized bed.

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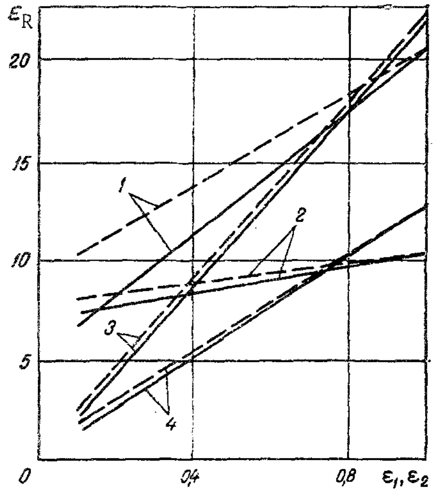

Fig. 2. Dependence of the resultant flux of thermal radiation (kW/m^2) absorbed by the fluidized bed on the emissivity of the radiator and of the surface of the fluidized bed. Dashed lines) flux calculated from Eq. (4); solid lines) flux calculated from Eq. (14a). 1, 2) Dependence on ε_1; 3, 4) dependence on ε_2, for H/D = 1.0 and 2.5, respectively.

In designing furnaces with radiative or convective–radiative heating of a bed, it is necessary to calculate the geometric dimensions of the furnace (the ratio of the furnace diameter to the distance from the radiating crown to the surface of the bed), under the assumption that all other parameters (the temperatures of the bed and the crown, the emissivity of the surfaces, etc.) are governed by the technology of the process. If the geometric dimensions of the furnace are chosen on the basis of structural considerations, it is necessary to carry out a verifying calculation to determine whether the necessary amount of heat can be transferred to the bed with the selected parameters of the technological process or if it is necessary to determine the temperature of the crown. We therefore need an equation which unambiguously gives us at least one of these parameters.

A furnace for the radiative heating of a fluidized bed is a closed system of several gray objects separated by a diathermic or absorbing medium which is in a state of radiative heat transfer. We need to find an equation for the resultant radiation flux absorbed by the surface of the fluidized bed. Let us determine the flux of thermal radiation from surface 1 to surface 2 (Fig. 1).

The amount of heat incident on surface 2 from surface 1 is given by the following equation for the case of a single reflection from each surface in the system:

$$Q_{\mathrm{inc}_1}^{1-2} = Q_1\varphi_{12} + Q_1\varphi_{13}r_3\varphi_{32} + Q_1\varphi_{13}r_3\varphi_{31}r_1\varphi_{12} + Q_1\varphi_{12}r_2\varphi_{21}r_1\varphi_{12} + Q_1\varphi_{12}r_2\varphi_{23}r_3\varphi_{32}, \quad (1)$$

where $Q_1 = E_1F_1$ is the heat flux associated with the radiation of surface 1 itself. Since the refractory lining materials used in practice (chamotte, Dianas brick, chrome–magnesite brick, high-alumina chamotte, etc.), as well as the fluidized beds, have a high emissivity ($\varepsilon = 0.8$-0.85), we can neglect the heat fluxes reflected from surfaces 1 and 2, corresponding to the third, fourth, and fifth terms in Eq. (1). We therefore assume that the effective radiation of the crown and the surface of the fluidized bed is equal to the intrinsic radiation. Calculations show that with $\varepsilon_1 = \varepsilon_2 = 0.8$ the error of this assumption does not exceed 5% for small values of H/D (H is the distance from the crown to the surface of the bed, and D is the furnace diameter), and for the ratios H/D = 1.0-4.0 used in practice this error does not exceed 1-2%.

Then we have

$$Q_{\mathrm{inc}_1}^{1-2} = E_1F_1(\varphi_{12} + \varphi_{13}\varphi_{32}r_3).$$

Surface 3, absorbing some of the radiant energy incident on it from surface 1, also radiates to surface 2. Using the assumption above, we can write the following equation for the flux of thermal radiation incident on surface 2 from surface 3, again for the case of single reflections:

$$Q_{\mathrm{inc}_1}^{3-2} = E_3F_3(\varphi_{32} + \varphi_{33}\varphi_{32}r_3).$$

Analogously, for the flux from surface 2 onto lateral surface 3, again for single reflections, we have

$$Q_{\mathrm{inc}_1}^{2-2} = E_2F_2(\varphi_{22} + \varphi_{23}\varphi_{32}r_3).$$

Now taking into account n reflections of the energy from surface 3, we write the flux from surface 1 to surface 2 as

TABLE 1. Resultant Flux of Thermal Radiation Absorbed by the Fluidized Bed

T_1, °K	T_2, °K	T_3, °K	ε_3	H/D	Resultant flux according to [4], W/m² resultant flux according to Eq. (14a), W/m² ε_1: 0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	
1223	573	773	0,8	1,0	7140	8511	10916	11498	13346	14782	16378	17895	19493	20985	ε_2=0,8
					9163	10615	11987	13329	14624	16034	17291	18597	19874	22431	
				2,5	8040	8245	8554	8745	9044	9295	9563	9825	10052	10346	
					8451	8599	8859	9085	9299	9572	9708	10032	10194	10395	
1673	973	1373		1,0	66906	70816	75512	80304	85115	89677	94664	99581	104376	108954	
					83567	85972	88614	91397	94134	97317	100617	102457	149738	107344	
				2,5	84126	84546	85514	84351	86644	87699	88683	89347	90010	90917	
					87959	88647	88532	89125	89515	89985	90424	90598	90971	91072	
					ε_2										
1223	573	773	0,8	1,0	2258	4461	6686	8986	11196	13381	15658	17944	21316	22791	ε_1=0,8
					2497	4812	7157	9477	11748	14175	16382	18076	20871	22896	
				2,5	1223	2455	3677	4979	6142	7349	8485	9855	11234	12334	
					1257	2528	3788	5052	6291	7514	8743	10032	11156	12381	
1673	973	1373		1,0	12315	24621	37191	49637	62145	74298	86671	99105	111495	124357	
					13567	27218	40919	53541	66534	79496	92169	10245	117216	128962	
				2,5	11134	22274	33424	44516	55986	66973	78394	89742	100671	111594	
					11579	22419	34519	45817	57148	68493	79522	90678	101644	112798	

$$Q_{\text{inc}}^{1-2} = E_1F_1(\varphi_{12} + \varphi_{13}r_3\varphi_{32} + \varphi_{13}r_3\varphi_{32}r_3\varphi_{33} + \cdots + \varphi_{13}r_3^n\varphi_{33}^{n-1}\varphi_{32}),$$

$$Q_{\text{inc}}^{1-2} = E_1F_1\left[\varphi_{12} + \varphi_{13}\varphi_{32}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n)\right]. \tag{2}$$

Here $E_1F_1\varphi_{12}$ is the flux incident on surface 2 without reflections, and $E_1F_1\varphi_{13}\varphi_{32}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n)$ is the flux from surface 1 to surface 2 as a result of the n-th reflection from surface 3 (N is an arbitrarily large number).

Analogously, we can write the flux which returns to surface 1 after the n-th reflection from surface 3:

$$Q_{\text{inc}}^{1-1} = \varphi_{13}\varphi_{31}\sum_{n-1}^{n=N}(\varphi_{33}^{n-1}r_3^n).$$

Part of the energy incident on surface 3 from surface 1 is absorbed by surface 3 in each reflection from it. This part of the energy is

$$Q_{\text{abs}}^{1-3} = E_1F_1\varphi_{13}(1-r_3) + E_1F_1\varphi_{13}r_3\varphi_{33}(1-r_3) + \cdots + E_1F_1\varphi_{13}(1-r_3)\varphi_{33}^{n-1}r_3^n;$$

$$Q_{\text{abs}}^{1-3} = E_1F_1\varphi_{13}\frac{1-r_3}{r_3}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n).$$

Defining

$$\frac{1-r_3}{r_3} = R_3,$$

we have

$$Q_{\text{abs}}^{1-3} = E_1F_1\varphi_{13}R_3\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n).$$

We obviously have

$$(Q_{\text{inc}}^{1-2} - Q_1\varphi_{12}) + Q_{\text{inc}}^{1-1} + Q_{\text{abs}}^{1-3} = Q_1\varphi_{13},$$

$$E_1F_1\varphi_{13}\varphi_{32}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n) + E_1F_1\varphi_{13}\varphi_{31}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n) + E_1F_1\varphi_{13}R_3\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n) = E_1F_1\varphi_{13}$$

and thus

$$\sum_{n=1}^{n=N}(\varphi_{33}^{n-1}r_3^n) = \frac{1}{\varphi_{32} + \varphi_{31} + R_3}. \tag{3}$$

Substituting (3) into (2) and carrying out some simple manipulations, we find

$$Q_{\text{inc}}^{1-2} = E_1F_1\left(\varphi_{12} + \frac{\varphi_{13}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}\right). \tag{4}$$

The flux from surface 3 to surface 2 after the n-th reflection from surface 3 is

$$Q_{\text{inc}}^{3-2} = E_3F_3(\varphi_{32} + \varphi_{33}r_3\varphi_{32} + \varphi_{33}r_3\varphi_{33}r_3\varphi_{32} + \cdots + \varphi_{32}\varphi_{33}^n r_3^n),$$

$$Q_{\text{inc}}^{3-2} = E_3F_3\varphi_{32}\sum_{n=0}^{n=N}(\varphi_{33}^n r_3^n). \tag{5}$$

Analogously, the flux from surface 3 to surface 1, with the n-th reflection taken into account, is

$$Q_{\text{inc}}^{3-1} = E_3F_3\varphi_{32}\sum_{n=0}^{n=N}(\varphi_{33}^n r_3^n).$$

The absorbed energy is

$$Q_{abs}^{3-3} = E_3F_3\varphi_{33}(1-r_3) + E_3F_3\varphi_{33}r_3\varphi_{33}(1-r_3) + \cdots$$
$$\cdots + E_3F_3(1-r_3)\varphi_{33}^{n} r_3^{n-1},$$
$$Q_{abs}^{3-3} = E_3F_3(1-r_3)\sum_{n=0}^{n=N}(\varphi_{33}^{n+1} r_3^n).$$

We note that

$$(1-r_3)\sum_{n=0}^{n=N}(\varphi_{33}^{n+1} r_3^n) = \left(\frac{1-r_3}{r_3}\right) r_3\varphi_{33}\sum_{n=0}^{n=N}(\varphi_{33}^{n} r_3^n).$$

Obviously, we have

$$Q_{inc}^{3-2} + Q_{inc}^{3-1} + Q_{abs}^{3-3} = E_3F_3$$

or

$$E_3F_3\varphi_{32}\sum_{n=0}^{n=N}(\varphi_{33}^n r_3^n) + E_3F_3\varphi_{31}\sum_{n=0}^{n=N}(\varphi_{33}^n r_3^n) + E_3F_3R_3r_3\varphi_{33}\sum_{n=0}^{n=N}(\varphi_{33}^n r_3^n) = E_3F_3,$$

and thus

$$\sum_{n=0}^{n=N}(\varphi_{33}^n r_3^n) = \frac{1}{\varphi_{32} + \varphi_{31} + R_3r_3\varphi_{33}}.$$

We then have

$$Q_{inc}^{3-2} = E_3F_3\frac{\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3r_3\varphi_{33}}. \tag{6}$$

The flux from surface 2 to surface 2 as a result of the n-th reflection from surface 3 is

$$Q_{inc}^{2-2} = E_2F_2\varphi_{23}r_3\varphi_{32} + E_2F_2\varphi_{23}r_3\varphi_{33}r_3\varphi_{32} + \cdots + E_2F_2\varphi_{33}^{n-1} r_3^n\varphi_{23}\varphi_{32}$$

or

$$Q_{inc}^{2-2} = E_2F_2\varphi_{23}\varphi_{32}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1} r_3^n). \tag{7}$$

Analogously, the flux from surface 2 to surface 1 as a result of the n-th reflection from surface 3 is

$$Q_{inc}^{2-1} = E_2F_2\varphi_{23}\varphi_{31}\sum_{n=1}^{n=N}(\varphi_{33}^{n-1} r_3^n). \tag{8}$$

The absorbed flux is

$$Q_{abs}^{2-3} = E_2F_2(1-r_3)\varphi_{23} + E_2F_2\varphi_{23}r_3\varphi_{33}(1-r_3) + \cdots + E_2F_2\varphi_{23}(1-r_3)\varphi_{33}^{n-1} r_3^{n-1}$$

or

$$Q_{abs}^{2-3} = E_2F_2\varphi_{23}R_3\sum_{n=1}^{n=N}(\varphi_{33}^{n-1} r_3^n). \tag{9}$$

We obviously have

$$Q_{inc}^{2-2} + Q_{inc}^{2-1} + Q_{abs}^{2-3} = E_2F_2\varphi_{23}. \tag{10}$$

Substituting (7)-(9) into (10), and carrying out certain simplifications, we find

$$\sum_{n=1}^{n=N}(\varphi_{33}^{n-1} r_3^n) = \frac{1}{\varphi_{32} + \varphi_{31} + R_3},$$

and thus

$$Q_{inc}^{2-2} = E_2F_2\frac{\varphi_{23}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}.$$

Taking into account the radiation from surface 2 to itself ($\varphi_{22} \neq 0$), we have

$$Q^{2-2}_{\text{inc}} = E_2 F_2 \frac{\varphi_{22} + \varphi_{23}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}. \tag{11}$$

Accordingly, the total heat flux incident on surface 2 as a result of radiative heat transfer in a system of three surfaces filled with a transparent medium, with the n-th reflection from the lateral surface taken into account, is

$$Q_{2\text{inc}} = Q^{1-2}_{\text{inc}} + Q^{2-2}_{\text{inc}} + Q^{3-2}_{\text{inc}},$$

$$Q_{2\text{inc}} = E_1 F_1 \left(\varphi_{12} + \frac{\varphi_{13}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}\right) + E_2 F_2 \left(\varphi_{22} + \frac{\varphi_{23}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}\right) + E_3 F_3 \frac{\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3 r_3 \varphi_{33}}. \tag{12}$$

Noting that the specific flux is $E_{2\text{inc}} = Q_{2\text{inc}}/F_2$, and that we have $d\varphi_{21}/\varphi_{12} = F_1/F_2$ and $\varphi_{23}/\varphi_{32} = F_3/F_2$, we find, from the reciprocity rule for angular coefficients,

$$E_{2_{R2}} = E_1 \left(\varphi_{21} + \frac{\varphi_{21}}{\varphi_{12}} \cdot \frac{\varphi_{13}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}\right) + E_2 \left(\varphi_{22} + \frac{\varphi_{23}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}\right) + E_3 \frac{\varphi_{23}}{\varphi_{32} + \varphi_{31} + R_3 r_3 \varphi_{33}}. \tag{13}$$

The resultant flux at surface 2 is

$$E_{R2} = E_{2\text{inc}}\varepsilon_2 - E_2,$$

so we have

$$E_{R2} = \left[E_1 \left(\varphi_{12} + \frac{\varphi_{13}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3} \cdot \frac{\varphi_{21}}{\varphi_{12}}\right) + E_2 \left(\varphi_{22} + \frac{\varphi_{23}\varphi_{32}}{\varphi_{32} + \varphi_{31} + R_3}\right) + E_3 \frac{\varphi_{23}}{\varphi_{32} + \varphi_{31} + R_3 r_3 \varphi_{33}}\right] \varepsilon_2 - E_2. \tag{14}$$

If

$$F_1 = F_2, \text{ then } \varphi_{12} = \varphi_{21}; \quad \varphi_{31} = \varphi_{32}; \quad \varphi_{23} = \varphi_{13}.$$

Assuming $\varphi_{22} = 0$, we find

$$E_{R2} = \left[E_1 \left(\varphi_{12} + \frac{\varphi_{23}\varphi_{32}}{2\varphi_{32} + R_3}\right) + E_2 \frac{\varphi_{23}\varphi_{32}}{2\varphi_{32} + R_3} + E_3 \frac{\varphi_{23}}{2\varphi_{32} + R_3 r_3 \varphi_{33}}\right] \varepsilon_2 - E_2. \tag{14a}$$

If $r_3 = 1$, then $E_3 = 0$, and the equation for E_{R2} becomes

$$E_{R2} = \varepsilon_2 \left(E_1 \frac{1 + \varphi_{12}}{2} + E_2 \frac{\varphi_{23}\varphi_{32}}{2}\right) - E_2. \tag{14b}$$

If $r_3 = 0$, then

$$E_{R2} = \varepsilon_2 (E_1 \varphi_{12} + E_3 \varphi_{23}) - E_2. \tag{14c}$$

We have thus derived quite simple equations for the specific flux of thermal radiation absorbed by the fluidized bed during radiative heating.

If the system is filled with an absorbing medium, its influence can be taken into account by a procedure analogous to that of [4].

It is interesting to compare the fluxes calculated from Eq. (14a) with those calculated from the equations of [4].

Table 1 shows the fluxes calculated from both equations for a system of a circular cylinder for the values $T_1 = 1223°K$, $T_2 = 573°K$, $T_3 = 773°K$, and $\varepsilon_3 = 0.8$ for two values of H/D, 1.0 and 2.5; and for $T_1 = 1673°K$, $T_2 = 973°K$, and $T_3 = 1373°K$, with the same values of H/D and with ε_1 and ε_2 varied from 0.1 to 1.0. Figure 2 shows a curve of the resultant heat flux as a function of the emissivity ε_1 and the emissivity of the heat-absorbing surface, ε_2. The numerical value of the emissivity of the surface of a fluidized bed can be determined from the equation given in [5].

In conclusion, we should point out that the calculations carried out on the basis of these equations agree satisfactorily with the experimental data of [6-8]. For example, with $\varepsilon_1 = \varepsilon_3 = 0.8$, $\varepsilon_2 = 0.9$, $T_1 = 1223°K$, $T_2 = 575°K$, $T_3 = 773°K$, and H/D = 1.0 and 1.55, the specific fluxes found in the experiments of [8] are 22.6 and 14.7 kW/m^2, while those calculated from Eq. (14a) are 20.9 and 15.1 kW/m^2.

NOTATION

$Q_{inc_1}^{i-k}$, flux of thermal radiation from surface i to surface k, with a single reflection from each surface in the system taken into account; Q_{inc}^{i-k}, the same, with an infinite number of reflections taken into account; φ_{ik}, angular coefficient from surface i to surface k; r_i, reflectivity of surface i; ε_i, emissivity of surface i; Q_i, heat flux of intrinsic radiation of surface i; E_i, intrinsic radiation of surface i; F_i, area of surface i.

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